# An Atoms in Molecules and Electron Localization Function Computational Study on the Molecular Structure of the 6-Tricyclo[3.2.1.0 ${ }^{2,4}$ ]octyl Cation 

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#### Abstract

The structure of 6-tricyclo[3.2.1.0 ${ }^{2,4}$ ] octyl cation 1 was optimized at HF, Becke3PW91, Becke3LYP, MP2, and MP2(full) levels with the $6-311 \mathrm{G}(\mathrm{d}, \mathrm{p})$ basis set. Becke3PW91 and MP2(full) yielded similar values for the geometrical parameters of optimized $\mathbf{1}$. When the $\mathrm{C} 4-\mathrm{C} 6$ distance was changed incrementally through the range $1.40-2.32 \AA$ at the Becke3PW91/6-311G(d,p) level, there was no discontinuity in the total energy and the geometrical parameters of the cyclopropyl group underwent marked changes. AIM (the theory of atoms in molecules) and ELF (electron localization function) analyses were carried out to investigate the molecular structure of $\mathbf{1}$. No bond path was found between C4 and C6 so $\mathbf{1}$ is a classical cation without pentacoordinated carbon atoms. Only when the C4-C6 distance was fixed in the range of $1.50-1.62 \AA$ does 1 become a pentacoordinate (nonclassical) species in which C4 is connected to C2, C3, C5, C6, and H13 with bond paths. On the other hand, when the 6 -tricyclo[3.2.1.0 ${ }^{2,4}$ ]octyl cation was substituted with Li and BeH groups at C 4 and C 5 , the optimized equilibrium species exhibted pentacoordinate carbons at C 4 . Compared with the $\mathrm{C} 1-\mathrm{C} 2$ bond that we take as a normal single bond, the bonds between C 4 and $\mathrm{C} 2, \mathrm{C} 3, \mathrm{C} 5$ are weak, as evidenced by reduced densities $\rho(\mathbf{r})$, significantly smaller $\nabla^{2} \rho(\mathbf{r})$ values, and large ellipticities at the bond critical points (BCPs). The delocalization indices (DIs)-the first application of a DI analysis in the study of carbocations-for these $\mathrm{C}-\mathrm{C}$ bonds ranged between 0.733 and 0.860 , smaller than unity. The DI between C 4 and C6 is 0.634 , suggesting that there is a significant degree of homoconjugation between C 4 and C 6 . For polycyclic species such as 1, it appears that a delocalization index of approximately 0.7 and an internuclear distance of $1.6 \AA$ are required for a bond path to materialize between remote carbons. In an ELF analysis we found a small disynaptic basin $\mathrm{V}(\mathrm{C} 4, \mathrm{C} 6)$ between C 4 and C 6 that correlated with the existence of a $(3,-1)$ critical point in $-\nabla^{2} \rho(\mathbf{r})$. The properties of ELF disynaptic basins around C4 and a contribution analysis showed a high degree of delocalization at this center.


## Introduction

Acetolysis of the exo,exo-, exo,endo-, endo,exo-6-tricyclo[3.2.1.0 ${ }^{2,4}$ ]octyl brosylates and nortricyclyl-3-carbinyl brosylate gives virtually identical mixtures of three acetates. A big difference in the solvolytic rates for the exo,exo-, and exo,endo6 -tricyclo[3.2.1.0 ${ }^{2,4}$ ]octyl brosylates led researchers to conclude that the 6 -tricyclo[3.2.1. $0^{2,4}$ ]octyl cation (1) is a so-called nonclassical species even though the exo,exo-6-tricyclo[3.2.1.0 ${ }^{2,4}$ ]octyl brosylate is less reactive by a factor of 0.15 than exo-2norbornyl brosylate. ${ }^{1,2}$ We have studied the 6 -tricyclo[3.2.1.0 ${ }^{2,4}$ ]octyl cation potential energy (PE) surface computationally and established that $\mathbf{1}$ is one of three common intermediates in the acetolysis of the brosylates and its molecular structure and rearrangement are important in determining the product compositions, the product distributions and the optical purities of the products. ${ }^{3}$ The cyclopropyl group has long been considered to be similar to a double bond and so can stabilize cations even at remote positions. ${ }^{4}$ It has played an important role in the development of the concept of homoaromaticity. ${ }^{5}$ The molecular structure of $\mathbf{1}$ is intriguing because a number of representations shown as $\mathbf{1 a}-\mathbf{1 e}^{2,4}$ were considered as possibilities for its molecular structure clearly indicating that there was some confusion about how to describe its bonding. If bond critical points can be located for the dashed bonds between C4 and C6 with AIM (the theory of atoms in molecules), then 1 can be considered a nonclassical species because C 4 is pentacoordinate



1d

1e



2
in these cases. Species 1a, 1c, 1d, and 1e can be viewed as unsymmetrical derivatives of pentacoordinate cation $\mathbf{2}$, one of the species on the $\mathrm{C}_{5} \mathrm{H}_{9}{ }^{+}$PE surface. ${ }^{6,7}$ On the other hand, it is possible that $\mathbf{1}$ is a $\pi / \pi$ or $\sigma / \pi$ no-bond homoconjugated species in which there is no bond path between C 4 and $\mathrm{C} 6 .{ }^{5}$

We showed in our previous publications that topologically based AIM and ELF (electron localization function) analyses are the methods of choice for studying so-called "nonclassical" species. ${ }^{8-11}$ In AIM, molecular space is partitioned through an analysis of the gradient vector field of the one-electron density $\rho(\mathbf{r})$ and a bond path is a line of maximum electron density between two nuclei. Any lateral displacement from this line leads to a decrease in $\rho(\mathbf{r}) .{ }^{12,13}$ Two nuclei are considered to be bonded when linked by a bond path. The integration of the oneelectron density within an atomic basin gives the atomic electron

TABLE 1: Selected Internuclear Distances ( $\AA$ ) of Cation 1 Optimized at Various Levels with the 6-311G(d,p) Basis Set

| level of theory | $\mathrm{C} 1-\mathrm{C} 2$ | $\mathrm{C} 2-\mathrm{C} 3$ | $\mathrm{C} 2-\mathrm{C} 4$ | $\mathrm{C} 3-\mathrm{C} 4$ | $\mathrm{C} 4-\mathrm{C} 5$ | $\mathrm{C} 4-\mathrm{C} 6$ | $\mathrm{C} 5-\mathrm{C} 6$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| HF | 1.530 | 1.409 | 1.637 | 1.645 | 1.555 | 1.692 | 1.442 |
| Becke3LYP | 1.538 | 1.427 | 1.623 | 1.644 | 1.566 | 1.726 |  |
| Becke3PW91 | 1.531 | 1.424 | 1.618 | 1.634 | 1.559 | 1.697 | 1.453 |
| MP2 | 1.530 | 1.431 | 1.616 | 1.645 | 1.562 | 1.690 | 1.451 |
| MP2(full) | 1.528 | 1.430 | 1.613 | 1.642 | 1.560 | 1.686 | 1.456 |

population, and consequently the net atomic charge. On the other hand, integration of the pair density within a single basin and between two basins will separately yield the localization index $\lambda(\mathrm{A}, \mathrm{A})=|F(\mathrm{~A}, \mathrm{~A})|$ and the delocalization index $\delta(\mathrm{A}, \mathrm{B})=$ $|F(\mathrm{~A}, \mathrm{~B})|+|F(\mathrm{~B}, \mathrm{~A})|{ }^{14-16}$

The other method we used involved a topological analysis of the gradient vector field of the Becke-Edgecombe electron localization function (ELF) ${ }^{17}$ as implemented by Silvi and coworkers. ${ }^{18}$ In ELF, core basins are organized around nuclei (with $Z>2$ ), providing an inner-atomic shell-like structure and valence basins occupy the remaining space. In the ELF picture, bonding is defined on the basis of how valence basins interact with core basins, that is the number of core basins a valence basin is stuck on. ${ }^{19}$ Numerous applications of ELF have demonstrated that a number of key points can be considered. A small basin population and a large basin fluctuation $\lambda(>0.5)$ indicates a high delocalization, and the contribution analysis can show how basins are delocalized. ${ }^{20}$ While there is usually a disynaptic basin between two bonded atoms, it has been suggested that "ionic" bonding is indicated if a basin is close to the core region of one of the atoms and no longer on the connection line. ${ }^{21}$ We have studied $\mathbf{1}$ with AIM and ELF and computed delocalization indexes (DIs)-the first application of DIs to the study of carbocations-with the goal of gaining information on the nature of the bonding at the $\mathrm{C} 4-\mathrm{C} 5-\mathrm{C} 6$ face of $\mathbf{1}$ and thereby establishing whether it is, in fact, best described as a pentacoordinate nonclassical species. Our findings and analyses are documented in this paper.

## Computational Details

Hartree-Fock (HF), DFT (Becke3PW91, Becke3LYP), and MP2 calculations were carried out with Gaussian $94^{22}$ on SGI Octane and SGI 2001 computers and with Gaussian $98^{23}$ on a Cray T90. The Becke3PW91/6-311G(d,p) level of theory was employed to obtain optimized geometries (Table 1) and wave functions throughout. As we established previously, calculations with the Becke3PW91 functional yielded geometries close to those obtained at the MP2(full) level with the same basis sets. ${ }^{11}$ Frequency calculations at the Becke3PW91/6-311G(d,p) level confirmed that cations 1 -opt, 3-opt, and 4-opt are minimum on the potential energy surface. The total energies in Table 2 (Supporting Information) do not include zero-point energies. At any rate, whether zero-point energies are included in studies of this type does not impact on the topological analyses. The studies of the electron density $\rho(\mathbf{r})$, its gradient vector field $\nabla \rho(\mathbf{r})$, and its Laplacian $\nabla^{2} \rho(\mathbf{r})$, as well as the integrations, were carried out with the AIMPAC suite of programs ${ }^{24}$ with the appropriate wave function. Electronic charge is locally concentrated in regions where $\nabla^{2} \rho(\mathbf{r})<0$. In the text, $L(\mathbf{r})$ refers to $-\nabla^{2} \rho(\mathbf{r})$ not $-\nabla^{2} \rho(\mathbf{r}) / 4$. We used the AIMDELOC program ${ }^{25}$ to calculate the localization ( $\lambda(\mathrm{A}, \mathrm{A})$ ) and delocalization indexes $(\delta(\mathrm{A}, \mathrm{B}))$ based on the Becke3PW91/6-311G(d,p) overlap matrix of each atom obtained from the AIMPAC integrations. We also obtained the LIs and DIs at the HF/6-311G(d,p)//Becke3PW91/ $6-311 \mathrm{G}(\mathrm{d}, \mathrm{p})$ and $\mathrm{HF} / 6-311 \mathrm{G}(\mathrm{d}, \mathrm{p}) / / \mathrm{HF} / 6-311 \mathrm{G}(\mathrm{d}, \mathrm{p})$ levels to establish how the DFT and HF LIs and DIs compare. The DFT
and HF results are virtually identical with the same basis set (Table 5). The ELF study was carried out with Silvi's TopMod package. ${ }^{26,27}$ A box size that extended 3.0 au from the outermost atomic coordinates in each direction and a step size of 0.1 au were typically used in these calculations. Additional details on the ELF calculations can be found in ref 11. The results were visualized with SciAn. ${ }^{28}$



## Results and Discussion

Energy and Geometry of the exo-6-Tricyclo[3.2.1.0 ${ }^{2,4}$ ]octyl Cation as a Function of the C4-C6 Distance. HF, Becke3PW91, Becke3LYP, MP2, and MP2(full) calculations (with the $6-311 \mathrm{G}(\mathrm{d}, \mathrm{p})$ basis set) yielded similar values for the geometrical parameters for optimized 1. At the Becke3LYP/6$311 \mathrm{G}(\mathrm{d}, \mathrm{p})$ level, the crucial C4-C6 distance (Table 1) was $1.726 \AA, 0.03 \AA$ longer than the average distance found with the other levels of theory. Unless specified otherwise, the following discussion is based on the results obtained at the Becke3PW91/6-311G(d,p) level. In going from tricyclo[3.2.1.0 ${ }^{2,4}$ ]octane, the parent hydrocarbon, to $\mathbf{1 - o p t}$, the $\mathrm{C} 4-\mathrm{C} 6$ internuclear distance decreased from 2.446 to $1.697 \AA$. There were other major changes. The $\mathrm{C} 2-\mathrm{C} 4, \mathrm{C} 3-\mathrm{C} 4$, and $\mathrm{C} 4-\mathrm{C} 5$ internuclear distances increased while the $\mathrm{C} 2-\mathrm{C} 3$ and $\mathrm{C} 5-\mathrm{C} 6$ distances decreased, suggesting that double-bond character developed at these centers. Table 2 that is included as Supporting Information lists selected geometry parameters, total energies, and relative energies of $\mathbf{1}$ as a function of the $\mathrm{C} 4-\mathrm{C} 6$ distance. Figure 1 shows the variation of the relative energy as a function of the $\mathrm{C} 4-\mathrm{C} 6$ distance. When the $\mathrm{C} 4-\mathrm{C} 6$ distance is decreased


Figure 1. Plot of the relative energies ( $\mathrm{kcal} \mathrm{mol}^{-1}$ ) of $\mathbf{1}$ as a function of the C4-C6 distance at the Becke3PW91/6-311G(d,p)/Becke3PW91/ 6-311G(d,p) level.
step-by-step from 1.697 to $1.40 \AA$ and the geometry optimized, the overall change in total energy was $12.67 \mathrm{kcal} \mathrm{mol}^{-1}$. The cyclopropyl group underwent large geometrical changes. The $\mathrm{C} 2-\mathrm{C} 4$ and $\mathrm{C} 3-\mathrm{C} 4$ distances increased by 0.229 and 0.120 A., respectively, while the $\mathrm{C} 2-\mathrm{C} 3$ distance shortened by 0.037 $\AA$. When the $\mathrm{C} 4-\mathrm{C} 6$ distance was increased from 1.697 to 2.320 $\AA$, the total energy increased by $10.55 \mathrm{kcal} \mathrm{mol}^{-1}$. At the $\mathrm{C} 4-$ C5-C6 face, the C4-C5 distance lengthened by $0.074 \AA$ as the C5-C6 distance shortened by $0.007 \AA$. In the case of the cyclopropyl group the $\mathrm{C} 2-\mathrm{C} 4$ and $\mathrm{C} 3-\mathrm{C} 4$ distances decreased by 0.107 and $0.114 \AA$, respectively, while the $\mathrm{C} 2-\mathrm{C} 3$ distance lengthened by $0.069 \AA$. When the $\mathrm{C} 4-\mathrm{C} 6$ distance reached 2.62 $\AA\left(\Delta E_{\mathrm{T}}=18.01 \mathrm{kcal} \mathrm{mol}^{-1}\right), 1$-opt rearranged to the isomeric cation 3-opt that is stabilized through participation of the C5C8 bond. In the whole range of $1.40-2.32 \AA$, there was no discontinuity in the total energy, and the other geometrical parameters also changed smoothly.

AIM Analysis and Delocalization Indexes. With the wave functions in hand, we analyzed the topology of charge density to gain information about the molecular structure of 1-opt. Figure 2 a is a contour plot of $\rho(\mathbf{r})$ in the $\mathrm{C} 4-\mathrm{C} 5-\mathrm{C} 6$ plane. A display of the gradient vector field of the charge density in the C4-C5-C6 plane is given in Figure 2b. There is no bond path between C4 and C6 even though the C4 and C6 basins appear to "impinge" on each other. This result is also seen in the 2-norbornyl cation that exhibits a T-structure regardless of whether Becke3LYP/6-31+G(d,p), MP2(full)/6-311G(d,p), or QCISD/6-31G(d,p) levels are used to obtain the wave function ${ }^{9-11}$ even though the $\mathrm{C} 1, \mathrm{C} 2$, and C 6 basins show this feature. Atomic interaction lines (bond paths) between C6 and C1 and C 2 were found for the 2-norbornyl cation only when the internuclear distances were fixed to less than $1.65 \AA$. Under these circumstances $\Delta E_{\mathrm{T}}$ increased by $>7 \mathrm{kcal} \mathrm{mol}^{-1}$, indicating that the species that is pentacoordinate at C 6 is not the minimum on the potential energy (PE) surface.

The properties at selected bond critical points for $\mathbf{1 - o p t}$ are given in Table 3. The C2-C4, C3-C4, and C4-C5 bonds are weak, as reflected in the small values of electron density $\rho(\mathbf{r})$ (1.194-1.431 e $\AA^{-3}$ ) and relatively small values of the Laplacian $\left(-1.856\right.$ to $\left.-7.639 \mathrm{e}^{\AA} \AA^{-5}\right)$. A contour plot of the Laplacian in the $\mathrm{C} 4-\mathrm{C} 5-\mathrm{C} 6$ plane is shown in Figure 2c. A $(3,-1)$ critical point in $L(\mathbf{r})$ at a value of 1.350 e $\AA^{-5}$ is located midway between C4 and C6, $0.871 \AA$ from C4 and $0.844 \AA$ from C6, and is labeled with an $\mathbf{x}$. As seen from the display of


Figure 2. Display of (a) the electron density in the C4-C5-C6 plane of 1-opt, (b) the gradient vector field of the electron density in the C4-C5-C6 plane, and (c) the Laplacian in the C4-C5-C6 plane. The bond CPs are labeled with circles. The $(3,-1) \mathrm{CP}$ in $L(\mathbf{r})$ is labeled with an $\mathbf{x}$.
the $0.675 \mathrm{e}^{\AA^{-3}}$ ( 0.1 au ) contour (Figure 3a), the density itself is not particularly enlightening, providing only an indication of the molecular shape. The plot of Laplacian at an iso value of

TABLE 3: Properties at the Bond Critical Points of 1

| bond | $\rho\left(\mathrm{e} \AA^{-3}\right)$ | $\nabla^{2} \rho\left(\mathrm{e} \AA^{-5}\right)$ | $\epsilon^{a}$ | $\lambda_{1}\left(\mathrm{e} \AA^{-5}\right)$ | $\lambda_{2}\left(\mathrm{e} \AA^{-5}\right)$ |
| :--- | :---: | :---: | ---: | ---: | ---: |
| $\mathrm{C} 1-\mathrm{C} 2$ | 1.633 | -13.038 | 0.011 | -10.700 | -10.604 |
| C2-H | 1.916 | -23.906 | 0.012 | -18.725 | -18.508 |
| C2-C3 | 1.930 | -17.327 | 0.192 | -13.640 | -11.447 |
| C2-C4 | 1.248 | -3.446 | 1.464 | -7.615 | -3.085 |
| C3-C4 | 1.194 | -1.856 | 2.975 | -7.182 | -1.807 |
| C4-H | 1.916 | -23.545 | 0.041 | -18.845 | -18.122 |
| C4-C5 | 1.431 | -7.639 | 0.500 | -9.134 | -6.097 |
| C5-C6 | 1.856 | -15.809 | 0.171 | -12.869 | -10.989 |
| C6-C7 | 1.721 | -14.532 | 0.016 | -11.471 | -11.278 |
| C6-H | 1.937 | -24.533 | 0.026 | -19.303 | -18.797 |

${ }^{a}$ The ellipticity $\epsilon$ is defined as $\lambda_{1} / \lambda_{2}-1$.


Figure 3. Display of (a) the $0.675 \mathrm{e}^{\AA^{-3}}(0.1 \mathrm{au})$ contour of the electron density of 1 -opt, (b) the $0.0 \mathrm{e}^{-5}$ contour of the Laplacian, (c) the $9.543 \mathrm{e} \AA^{-5}$ contour of the Laplacian around C 4 , and (d) the $(3,-3)$ CPs (yellow spheres) of the VSCCs located around C4 and C6 (blue spheres).
0.0 e $\AA^{-5}$ displayed in Figure 3 b confirmed that a saddle point might be expected between C 4 and C 6 . Of particular interest is the fact that C4 exhibited only four VSCCs in $L(\mathbf{r})$ (the $(3,-3)$ CPs lie $0.521-0.532 \AA$ from the nucleus) that are directed toward C2, C3, C5, and H13, but not at C6 (compare the contours between $\mathrm{C} 4-\mathrm{C} 5$ and $\mathrm{C} 4-\mathrm{C} 6$ of Figure 2c). While most covalent bonds exhibiting bond paths have two VSCC maxima, in some cases only one maximum occurs, as, for example, in the polarized $\mathrm{C}-\mathrm{F}$ bond, or none are seen, as in the case of the $\mathrm{P}-\mathrm{F}$ bond in $\mathrm{PF}_{3} .{ }^{29}$ The nature of the Laplacian around C 4 at an iso value of 9.543 e $\AA^{-5}$ is displayed in Figure 3c; Figure

3d shows the location of the $(3,-3) \mathrm{CPs}$, indicating how VSCCs are orientated around C 4 and directed toward the other atoms. The $\mathrm{C} 4-\mathrm{VSCC}-\mathrm{C} 2, \mathrm{C} 4-\mathrm{VSCC}-\mathrm{C} 3, \mathrm{C} 4-\mathrm{VSCC}-\mathrm{C} 3$, and $\mathrm{C} 4-\mathrm{VSCC}-\mathrm{H}$ angles are $153.1^{\circ}, 157.9^{\circ}, 158.0^{\circ}$, and $179.6^{\circ}$, respectively, indicating that the VSCCs try to keep as far as possible away from each other. It is interesting to note that C6, nominally considered to be tricoordinate, exhibits four VSCCs.
The net atomic charges of 1-opt are listed in Table 4. For C3, C4, and C6 the net charges are $+0.021,-0.211$, and +0.026 , respectively, at the Becke3PW91/6-311G(d,p)// Becke3PW91/6-311G(d,p) level. That C4 bears an unexpectedly large negative charge (at the HF/6-311G(d,p)//Becke3PW91/ $6-311 \mathrm{G}(\mathrm{d}, \mathrm{p})$ and $\mathrm{HF} / 6-311 \mathrm{G}(\mathrm{d}, \mathrm{p}) / / \mathrm{HF} / 6-311 \mathrm{G}(\mathrm{d}, \mathrm{p})$ levels, the charges are enormous ( -0.339 and -0.323 e , respectively)) is noteworthy and an indication that there is an "ionic" component to the $\mathrm{C} 4-\mathrm{C} 6$ interaction. We also integrated the pair density within single basins and between basins to obtain the localization and delocalization indexes $\lambda(\mathrm{A}, \mathrm{A})(\mathrm{LI})$ and $\delta(\mathrm{A}, \mathrm{B})(\mathrm{DI})$ that are given in Table 5. The total number of electrons in $\mathbf{1}$ is 58.0, and the sum of all LIs and DIs gave a value of 57.978, indicating that the integrations were of good quality. The extent of spatial localization of electron pairs is determined by the corresponding property of the Fermi hole density. Integration of the pair density yields the Fermi correlation contained in a single basin $F(\mathrm{~A}, \mathrm{~A})$ (its magnitude is the localization index $\lambda(\mathrm{A}, \mathrm{A})$ ) and $F(\mathrm{~A}, \mathrm{~B})$, the correlation shared between two basins. The quanity $F(\mathrm{~A}, \mathrm{~B})$ is a measure of the extent to which electrons of either spin referenced to atom A are delocalized into atom B with a corresponding definition of $F(\mathrm{~B}, \mathrm{~A})$. Thus $F(\mathrm{~A}, \mathrm{~B})=F(\mathrm{~B}, \mathrm{~A})$ and their sum $|F(\mathrm{~A}, \mathrm{~B})|+|F(\mathrm{~B}, \mathrm{~A})|=\delta(\mathrm{A}, \mathrm{B})$ that is termed the delocalization index is a measure of the total Fermi correlation shared between the atoms (basins). According to the integration property of the Fermi hole of an electron, the atomic and interatomic Fermi correlations sum to $-N$, the total number of electrons of a molecule. Correspondingly, the sum of the localization and delocalization indexes gives $N$ while providing a quantitative measure of how the $N$ electrons are localized within the atomic basins and delocalized between them. Each atomic population $\bar{N}(\mathrm{~A})$ is given by the expression $\bar{N}(\mathrm{~A})=$ $|F(\mathrm{~A}, \mathrm{~A})|+\sum_{\mathrm{B} \neq \mathrm{A}} \delta(\mathrm{A}, \mathrm{B}) / 2$. The localization index $|F(\mathrm{~A}, \mathrm{~A})|$ has a limiting value of $\bar{N}(\mathrm{~A})$, corresponding to complete localization of the $\bar{N}(\mathrm{~A})$ electrons to the basin of atom A . The percent localization (PL) given as $|F(\mathrm{~A}, \mathrm{~A})| / \bar{N}(\mathrm{~A}) \times 100$ is a measure of the electron localization and provides information about the ionic character of a bond; if the value is in the region of $100 \%$, the population is -1.0 or +1.0 , for example, bonding is purely ionic. As seen in Table 5, the PLs for C2, C3, C4, C5, and C6 of 1 that lie in the region of $66 \%$ do not differ significantly from the PLs of the parent hydrocarbon and they are virtually

TABLE 4: Net Charge on Carbon Atoms as a Function of the C4-C6 Distance and BeH Substitution ${ }^{a}$

|  | C4-C6 distance/Å |  |  |  |  | 4,5-(BeH) $)_{2}$ opt |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1.40 | 1.55 | 1.62 | $1.697(1.692)^{\text {b,c }} \mathrm{opt}$ | 2.32 |  |
| C1 | 0.029 | 0.038 | 0.040 | 0.043 (0.123) [0.118] | -0.051 | 0.043 |
| C2 | 0.015 | 0.012 | 0.013 | 0.011 (0.041) [0.037] | -0.019 | -0.0003 |
| C3 | 0.007 | 0.024 | 0.023 | 0.021 (0.104) [0.093] | -0.005 | 0.017 |
| C4 | -0.205 | -0.221 | -0.219 | -0.211 (-0.323) [-0.339] | -0.089 | -0.855 |
| C5 | 0.006 | 0.006 | 0.007 | 0.008 (0.029) [0.065] | 0.005 | -0.673 |
| C6 | 0.048 | 0.036 | 0.031 | 0.026 (0.075) [0.023] | 0.026 | 0.019 |
| C7 | 0.032 | 0.032 | 0.032 | 0.030 (0.157) [0.146] | 0.025 | 0.028 |
| C8 | 0.023 | 0.019 | 0.019 | 0.018 (0.135) [0.124] | -0.016 | 0.006 |

${ }^{a}$ Unless otherwise noted, the calculation level is Becke3PW91/6-311G(d,p)//Becke3PW91/6-311G(d,p). ${ }^{b}$ The values in parentheses were obtained with the wave function obtained at the $\mathrm{HF} / 6-311 \mathrm{G}(\mathrm{d}, \mathrm{p}) / / \mathrm{HF} / 6-311 \mathrm{G}(\mathrm{d}, \mathrm{p})$ level. The values in square brackets were obtained with the wave function obtained at the $\mathrm{HF} / 6-311 \mathrm{G}(\mathrm{d}, \mathrm{p}) / /$ Becke3PW91/6-311G(d,p) level. ${ }^{c}$ The corresponding values for C 1 to C 8 for the parent hydrocarbon tricyclo[3.2.1. $0^{2,4}$ ]octane are $0.046,-0.046,-0.051,-0.046,0.046,0.014,0.014$ and 0.003 , respectively.

TABLE 5: Localization Indexes $|\boldsymbol{F}(\mathbf{A}, \mathrm{A})|$ of Selected Atomic Basins and Delocalization Indexes $\boldsymbol{\delta}(\mathrm{A}, \mathrm{B})$ of Selected Pairs of Basins ${ }^{a}$

|  | A | \|F(A,A)| | $\%^{b}$ | A,B | $\delta(\mathrm{A}, \mathrm{B})$ | C4-C6 | A | \|F(A,A)| | $\%^{b}$ | A, B | $\delta(\mathrm{A}, \mathrm{B})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{opt}^{\text {c }}$ | C1 | 3.864 | 64.9 | C1, C2 | 0.944 (0.937) ${ }^{\text {d }}$ | 1.55 | C1 | 3.869 | 64.9 | C1,C2 | 0.950 |
|  | C2 | 3.918 | 65.4 | C2, C3 | 1.112 (1.108) |  | C2 | 3.929 | 65.6 | C2, C3 | 1.1670 |
|  | C3 | 3.983 | 66.6 | C2, C4 | 0.745 (0.742) |  | C3 | 3.989 | 66.8 | C2,C4 | 0.660 |
|  | C4 | 4.105 | 66.1 | C3, C4 | 0.733 (0.731) |  | C4 | 4.105 | 66.0 | C3, C4 | 0.673 |
|  | C5 | 3.913 | 65.3 | C4, C5 | 0.860 (0.862) |  | C5 | 3.924 | 65.5 | C4, 55 | 0.875 |
|  | C6 | 3.945 | 66.1 | C4,C6 | 0.634 (0.626) |  | C6 | 3.903 | 65.4 | C4, 66 | 0.784 |
|  | $\mathrm{H}(\mathrm{C} 4)$ | 0.333 | 38.6 | C5,C6 | 1.037 (1.036) |  | $\mathrm{H}(\mathrm{C} 4)$ | 0.330 | 38.4 | C5,C6 | 0.998 |
|  |  |  |  | C4, H | 0.911 (0.935) |  |  |  |  | C4, H | 0.905 |
| HF opt ${ }^{e}$ | C1 | 3.801 | 64.7 | C1,C2 | 0.937 | 1.62 | C1 | 3.867 | 64.9 | C1,C2 | 0.947 |
|  | C2 | 3.908 | 65.6 | C2, C3 | 1.124 |  | C2 | 3.923 | 65.5 | C2, C3 | 1.138 |
|  | C3 | 3.909 | 66.3 | C2, C4 | 0.717 |  | C3 | 3.986 | 66.7 | C2, C4 | 0.703 |
|  | C4 | 4.220 | 66.8 | C3, C4 | 0.720 |  | C4 | 4.106 | 66.0 | C3, C4 | 0.703 |
|  | C5 | 3.858 | 65.1 | C4, C5 | 0.869 |  | C5 | 3.918 | 65.4 | C4, 55 | 0.867 |
|  | C6 | 3.967 | 66.4 | C4,C6 | 0.636 |  | C6 | 3.923 | 65.7 | C4, 66 | 0.711 |
|  | $\mathrm{H}(\mathrm{C} 4)$ | 0.357 | 39.9 | C5,C6 | 1.036 |  | $\mathrm{H}(\mathrm{C} 4)$ | 0.330 | 38,0.4 | C5,C6 | 1.018 |
|  |  |  |  | C4, H | 0.939 |  |  |  |  | C4, H | 0.908 |
| 4,5-(BeH) $2_{2}$ | C1 | 3.860 | 64.8 | C1,C2 | 0.959 | 2.32 | C1 | 3.856 | 64.8 | C1,C2 | 0.943 |
|  | C2 | 3.919 | 65.3 | C2, C3 | 1.122 |  | C2 | 3.933 | 65.4 | C2,C3 | 0.984 |
|  | C3 | 3.969 | 66.4 | C2, C4 | 0.740 |  | C3 | 3.977 | 66.2 | C2, C4 | 0.946 |
|  | C4 | 4.925 | 71.9 | C3, C4 | 0.768 |  | C4 | 4.018 | 66.0 | C3, 44 | 0.928 |
|  | C5 | 4.803 | 72.0 | C4, 55 | 0.912 |  | C5 | 3.908 | 65.2 | C4, 55 | 0.821 |
|  | C6 | 3.916 | 65.5 | C4,C6 | 0.727 |  | C6 | 4.053 | 67.9 | C4, 66 | 0.189 |
|  | Be13 | 2.01 | 85.9 | C5,C6 | 1.068 |  | $\mathrm{H}(\mathrm{C} 4)$ | 0.366 | 40.5 | C5,C6 | 1.122 |
|  | Be14 | 2.01 | 86.1 | C4,Be13 | 0.252 |  |  |  |  | C4, H | 0.936 |
|  | H20 | 1.552 | 86.9 | C5,Be14 | 0.259 |  |  |  |  |  |  |
|  | H21 | 1.567 | 87.3 | Be13,H | 0.351 |  |  |  |  |  |  |
|  |  |  |  | Be14,H | 0.346 |  |  |  |  |  |  |

[^0]identical at the DFT and HF levels. There was no significant dependence of the PLs on the $\mathrm{C} 4-\mathrm{C} 6$ distance.

To ascertain the effect of substituents on the molecular structure of $\mathbf{1}$, we replaced the hydrogens of $\mathbf{1}$ with a range of substituents including $\mathrm{Li}, \mathrm{BeH}, \mathrm{BH}_{2}, \mathrm{CH}_{3}$, and $\mathrm{SiH}_{3}$. Only when Li and BeH groups replace hydrogens on C 4 and C 5 (the $\mathrm{C} 4-$ C6 distances are 1.617 and $1.660 \AA$ ) was a bond path found between C4 and C6 with five VSCCs located around C4. As was seen in 1-opt when the $\mathrm{C} 4-\mathrm{C} 6$ distance was fixed at 1.62 $\AA$, the bond path dwells away from atomic connection line and the bond $\mathrm{CP}(\mathbf{o})$ is very close to the ring $\mathrm{CP}(*)$ (Figure 6a), an indication that these CPs are close to the point of anihilation. Moreover, the electron densities at the bond and the ring CPs are equal ( $1.221 \mathrm{e}^{-3}$ ), and the Laplacian has values of -0.868 and 1.078 e $\AA^{-5}$, respectively. Only when the hydrogens at C4 and C5 are replaced with the electropositive substituents BeH and Li (data not shown for the latter) does the ionicity of the bonding to C 4 and C 5 increase and do these carbon atoms possess large negative charges (Table 4).

According to theory, the $1,2-\mathrm{DI}$ is unity when a single pair of electrons is shared equally between adjacent atoms of a homonuclear bond. ${ }^{16,17}$ The 1,3-DI between the C 4 and C 6 basins is surprisingly large ( 0.634 ), but smaller than the 1,2-DIs between $\mathrm{C} 2-\mathrm{C} 4$ ( 0.745 ), $\mathrm{C} 3-\mathrm{C} 4$ ( 0.733 ), and $\mathrm{C} 4-\mathrm{C} 5$ ( 0.860 ), which, unlike C 4 and C 5 , are connected by bond paths. In the cases $\mathbf{1}$ with the $\mathrm{C} 4-\mathrm{C} 6$ distance fixed at $1.62 \AA$ and $\mathbf{1}$ substituted with BeH and Li groups at C 4 and C 5 , where bond paths connected C 4 and C6, the 1,3 DIs increased to 0.711 and 0.727 , respectively. For the parent hydrocarbon and $\mathbf{1}$ with the C4C6 distance fixed at $2.32 \AA$ the C4 and C6 DIs are 0.057 and 0.189 , respectively, indicating that there is little or no homoconjugative interaction between C4 and C6 in these cases. These
results suggest that a 1,3 DI of greater than 0.7 and an internuclear distance in the range $1.6-1.65 \AA$ are the key requirements for the existence of pentacoordinate carbons in polycyclic carbocations. The DIs between C 4 and other atomic basins of $\mathbf{1}$ sum to 4.214 , roughly obeying the octet rule (four pairs, eight electrons). The DIs of $\mathrm{C} 2-\mathrm{C} 3$ and $\mathrm{C} 5-\mathrm{C} 6$ are greater than 1.00 ( 1.037 and 1.112) in keeping with the fact that these bonds have double bond character. To relate the DIs of a classical cation that is considered to be stabilized by hyperconjugation, we studied the $C_{3 v}$ tert-butyl cation (4-opt) at the Becke3PW91/6-311G(d,p) level. The H6-C2, H5-C2, and $\mathrm{H} 7-\mathrm{C} 21,3-\mathrm{DIs}$ are $0.101,0.052$, and 0.052 , respectively; the $\mathrm{C}-\mathrm{H}$ distances for these two different types of hydrogens are 2.049 and $2.141 \AA$. That hyperconjugation reduces the degree of covalent bonding in the $\mathrm{C}-\mathrm{H}$ bond that is coplanar with the vacant p orbital is seen in the fact that the $\mathrm{C} 1-\mathrm{H} 6 \mathrm{DI}$ ( 0.865 ) is less than the $\mathrm{C} 1-\mathrm{H} 5$ and $\mathrm{C} 1-\mathrm{H} 7$ DIs ( 0.921 ). For the parent hydrocarbon, 2-methylpropane, the 1,3-DIs between C2 and the methyl hydrogens (the distances are 2.181 and 2.174 $\AA$ ) are $0.0393,0.0393$, and 0.041 , respectively, and the DIs for the $\mathrm{C}-\mathrm{H}$ bonds are $0.958,0.958$, and 0.950 . We take these as standard values for the isobutyl system. Thus, the 1,3 -DIs for the tert-butyl cation are only marginally larger than the values found for the parent hydrocarbon and it is clear that the internuclear distance/bond angle may be one of the factors that determine the magnitude of the DI. It is also interesting to note that the $\mathrm{C} 1-\mathrm{C} 2 \mathrm{DI}$ is 1.111 , confirming, as expected, that there is double bond character (the $\mathrm{C}-\mathrm{C}$ distance is $1.460 \AA$ ) in the three $\mathrm{C}-\mathrm{C}$ bonds of 4 -opt. For the parent hydrocarbon, the DI for the $\mathrm{C}-\mathrm{C}$ bond $(1.529 \AA$ ) is 0.981 . The results obtained for 4-opt are in keeping with the generally accepted view that hyperconjugation is maximized when a $\mathrm{C}-\mathrm{H}$ bond is aligned
coplanar with an adjacent vacant p orbital on carbon. That the $\mathrm{C}-\mathrm{Be}$ and $\mathrm{Be}-\mathrm{H}$ bonds of $4,5-(\mathrm{BeH})_{2}-\mathbf{1}$ have a large degree of ionicity is seen in their DIs: 0.259 and 0.252 for the former and 0.346 and 0.351 for the latter.

ELF Analysis. Our ELF study on the 2-norbornyl cation ${ }^{12}$ at the Becke3LYP/cc-pVTZ level revealed that a single trisynaptic basin connected C6 to C1 and C2 in the optimized geometry, while disynaptic basins linked $\mathrm{C} 1-\mathrm{C} 6$ and $\mathrm{C} 2-\mathrm{C} 6$ at the MP2(full)/6-311G(d,p), Becke3PW91/cc-pVTZ, and QCISD/6-31G(d,p) levels. While the C6-C1(C2) distance was $1.889 \AA$ at Becke3LYP it ranged between 1.826 and $1.846 \AA$ at the MP2(full), QCISD, and Becke3PW91 levels. Yet small increases in the $\mathrm{C} 6-\mathrm{C} 1(2)$ distances resulted in C 6 being disconnected from C 1 and C 2 with $\Delta E_{\mathrm{T}}$ ranging from 0.14 to $0.50 \mathrm{kcal} \mathrm{mol}^{-1}$ depending on the level of theory used. No discontinuity was seen in $E_{\mathrm{T}}$ when these disconnections occurred. This study established that small changes in molecular geometry affect the outcome of ELF analyses so attention was paid to that possibility. The integration of the one-electron density $\rho(\mathbf{r})$ and of the pair function $\pi\left(\mathbf{r}_{1}, \mathbf{r}_{2}\right)$ over the volume $\Omega$ of one basin provides the population $\bar{N}(\Omega)$ of a basin and its variance $\sigma^{2}$, the latter being a measure of the contribution from other basins to $\bar{N}(\Omega)$. The relative fluctuation $\lambda=\sigma^{2} / \bar{N}(\Omega)$ provides an indication of the delocalization within that basin, while the percent contribution fluctuations (\%) give a quantitative measure of the contributions of other basins to the variance $\sigma^{2} .{ }^{19}$

The $\mathrm{C}-\mathrm{C}$ valence basins of 1-opt can basically be divided into three groups. The first group that includes $\mathrm{V}(\mathrm{C} 2, \mathrm{C} 4), \mathrm{V}(\mathrm{C} 3,-$ $\mathrm{C} 4)$, and $\mathrm{V}(\mathrm{C} 4, \mathrm{C} 6)$ has small populations ( $0.83-1.22 \mathrm{e}$ ), small volumes ( $10.28-12.60 \AA^{3}$ ), and large fluctuations ( $0.68-0.76$ ). They are also differentiated by the fact that the attractors, the $(3,-3) \mathrm{CPs}$, are located closer to $\mathrm{C} 4(0.777-0.793 \AA)$ than the other carbon atoms $(0.883-1.002 \AA)$ and they do not lie on the geometric connection line between the atoms. The C4$\mathrm{V}(\mathrm{C} 4, \mathrm{C})-\mathrm{C}$ angles range between $144.8^{\circ}$ and $153.6^{\circ}$. The second group includes $\mathrm{V}(\mathrm{C} 5, \mathrm{C} 6)$ and $\mathrm{V}(\mathrm{C} 2, \mathrm{C} 3)$ that have a teardrop shape at the 0.75 contour (Figure $4 \mathrm{c}(4 \mathrm{~d})$ and Figure $4 \mathrm{e}(4 \mathrm{f})$ in yellow) in keeping with the fact that these bonds exhibit partial double bond character; ethene exhibits a dumbbell-shaped C,C valence basin at this contour value. The $\mathrm{V}(\mathrm{C} 2, \mathrm{C} 3) \mathrm{CP}$ is equidistant from C 2 and C 3 , while the one associated with $\mathrm{V}(\mathrm{C} 5, \mathrm{C} 6)$ is slightly closer to C6 $(0.743 \AA)$ than C5 $(0.791 \AA)$. The remaining basins that are pill-shaped (in green) fall in the third group. The CPs lie on the geometric atomic connection line between the carbon atoms and their basin populations are in the region 2.0 , showing the normal covalent $\mathrm{C}-\mathrm{C}$ bonds within the ELF formalism. V(C4,C5) exhibits properties (a population of 1.64 e , a volume $20.4 \AA^{3}$, and a fluctuation of 0.60 ) that lie roughly midway between those of these basins and the first group. The disynaptic basin $\mathrm{V}(\mathrm{C} 4, \mathrm{C} 6)$ between C4 and C6, shown in red in Figure 4a,c) has an ELF value of 0.810 at its $(3,-3) \mathrm{CP}$ (attractor). It has a small population of only 0.83 e , a large fluctuation $(0.76)$, and its $(3,-3) \mathrm{CP}$ (yellow) is located $0.771 \AA$ from C4 and $1.000 \AA$ from C6, as seen in Figure 4 b ; the $\mathrm{C} 4-\mathrm{V}(\mathrm{C} 4, \mathrm{C} 6)-\mathrm{C} 6$ angle is $144.8^{\circ}$. As seen in Table 6, seven valence basins communicate (percent contributions of less than 4\% are arbitrarily excluded) with the C4-C6 basin of 1-opt.

Silvi and co-workers have demonstrated that localization reduction tree diagrams are useful in analyzing the hierarchy of valence basins. ${ }^{30}$ The tree diagram for $\mathbf{1 - o p t}$ shows that it is possible to divide the valence shell of C 4 (see Figure 4a) roughly into three main regions (a) $\mathrm{V}(\mathrm{C} 4, \mathrm{H})$, (b) the union of the $\mathrm{V}(\mathrm{C} 3,-$


Figure 4. Display of (a) the 0.75 contour of the ELF basins of 1-opt with the following color coding: C,H basins, light blue; $\mathrm{C} 3, \mathrm{C} 4$ basin, purple; C4,C2 basin, orange; C4,C5 basin, magenta; C4,C6 basin, red; $\mathrm{C} 2, \mathrm{C} 3$ and $\mathrm{C} 5, \mathrm{C} 6$ basins, yellow; $\mathrm{C} 1, \mathrm{C} 2, \mathrm{C} 6, \mathrm{C} 7$, and $\mathrm{C} 7, \mathrm{C} 1$ basins, green; carbon cores, small purple spheres, (b) the location of the (3, -3) CPs (attractors) in ELF around C4 and C6, (c) the ELF basins ( 0.75 contour) viewed perpendicular to the $\mathrm{C} 4-\mathrm{C} 5-\mathrm{C} 6$ plane, (d) 1-opt viewed perpendicular to the $\mathrm{C} 4-\mathrm{C} 5-\mathrm{C} 6$ plane, (e) the ELF basins $(0.75$ contour) viewed perpendicular to the $\mathrm{C} 2-\mathrm{C} 3-\mathrm{C} 4$ plane, (f) 1-opt viewed perpendicular to the $\mathrm{C} 2-\mathrm{C} 3-\mathrm{C} 4$ plane.
$\mathrm{C} 4)$ and $\mathrm{V}(\mathrm{C} 4, \mathrm{C} 5)$ basins and (c) the union of $\mathrm{V}(\mathrm{C} 2, \mathrm{C} 4)$ and $\mathrm{V}(\mathrm{C} 4, \mathrm{C} 6)$ basins. The bifurcation values for the union of the $\mathrm{V}(\mathrm{C} 3, \mathrm{C} 4)$ and $\mathrm{V}(\mathrm{C} 4, \mathrm{C} 5)$ basins and the union of $\mathrm{V}(\mathrm{C} 2, \mathrm{C} 4)$ and $\mathrm{V}(\mathrm{C} 4, \mathrm{C} 6)$ are 0.771 and 0.786 , and so they still share separate matrices at an ELF value of 0.75, as seen in Figure 4a. Nevertheless, the ELF values at the attractors of the four disynaptic basins $\mathrm{V}(\mathrm{C} 3, \mathrm{C} 4), \mathrm{V}(\mathrm{C} 4, \mathrm{C} 5), \mathrm{V}(\mathrm{C} 2, \mathrm{C} 4)$, and $\mathrm{V}(\mathrm{C} 4,-$ C6) are $0.856,0.915,0.867$, and 0.810 , respectively. Considering


TABLE 6: Population $(\bar{N})$, Variance ( $\boldsymbol{\sigma}^{2}$ ), Relative Fluctuation ( $\boldsymbol{\lambda}$ ), Volume ( $\boldsymbol{\Omega}$, Bohr ${ }^{3}$ ), ELF Values $(\boldsymbol{\eta})$ at $(\mathbf{3},-\mathbf{3})$ Critical Points and Contribution of Other Basins to the Variance ( $\sigma^{2}$ ) of Selected Basins of 1

| basin | $\bar{N}$ | $\sigma^{2}$ | $\lambda$ | $\Omega$ | $h$ | contribution analysis (\%) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| V(C1, C2) | 1.93 | 1.02 | 0.53 | 19.52 | 0.957 | $15.0 \mathrm{~V}(\mathrm{H}, \mathrm{C} 1) ; 14.7 \mathrm{~V}(\mathrm{H}, \mathrm{C} 2) ; 13.1 \mathrm{~V}(\mathrm{C} 2, \mathrm{C} 3) ; 8.5 \mathrm{~V}(\mathrm{C} 2, \mathrm{C} 4) ; 5.9 \mathrm{C}(\mathrm{C} 2) ; 4.8 \mathrm{C}(\mathrm{C} 1)$ |
| $\mathrm{V}(\mathrm{C} 2, \mathrm{H})$ | 2.12 | 0.67 | 0.32 | 72.69 | 1.000 | $\begin{aligned} & 24.7 \mathrm{~V}(\mathrm{C} 2, \mathrm{C} 3) ; 22.8 \mathrm{~V}(\mathrm{C} 1, \mathrm{C} 2) ; 15.3 \mathrm{~V}(\mathrm{C} 2, \mathrm{C} 4) ; 11.2 \mathrm{C}(\mathrm{C} 2) ; 3.0 \mathrm{~V}(\mathrm{H} 12, \mathrm{C} 3) ; 2.5 \mathrm{~V}(\mathrm{H}, \mathrm{C} 4) \text {; } \\ & \text { 2.4V(H11,C3); } \end{aligned}$ |
| $\mathrm{V}(\mathrm{C} 2, \mathrm{C} 3)$ | 2.09 | 1.06 | 0.51 | 32.33 | 0.949 | $\begin{aligned} & 15.7 \mathrm{~V}(\mathrm{H} 11, \mathrm{C} 3) ; 15.6 \mathrm{~V}(\mathrm{H}, \mathrm{C} 2) ; 15.5 \mathrm{~V}(\mathrm{H} 12, \mathrm{C} 3) ; 10.1 \mathrm{~V}(\mathrm{C} 4 . \mathrm{C} 6) ; 9.3 \mathrm{~V}(\mathrm{C} 3, \mathrm{C} 4) ; 5.9 \mathrm{C}(\mathrm{C} 2) ; \\ & 5.9 \mathrm{C}(\mathrm{C} 3) \end{aligned}$ |
| $\mathrm{V}(\mathrm{C} 2, \mathrm{C} 4)$ | 1.22 | 0.82 | 0.68 | 12.60 | 0.867 | $\begin{aligned} & 13.0 \mathrm{~V}(\mathrm{C} 2, \mathrm{C} 3) ; 12.3 \mathrm{~V}(\mathrm{H}, \mathrm{C} 2) ; 11.7 \mathrm{~V}(\mathrm{H}, \mathrm{C} 4) ; 11.3 \mathrm{~V}(\mathrm{C} 4, \mathrm{C} 6) ; 11.3 \mathrm{~V}(\mathrm{C} 3, \mathrm{C} 4) ; 10.4 \mathrm{~V}(\mathrm{C} 1, \mathrm{C} 2) \text {; } \\ & 4.4 \mathrm{C}(\mathrm{C}) ; 3.0 \mathrm{C}(\mathrm{C} 2) ; \end{aligned}$ |
| V(C3, 4 4) | 1.04 | 0.74 | 0.71 | 11.68 | 0.856 | $\begin{aligned} & 14.0 \mathrm{~V}(\mathrm{H}, \mathrm{C} 4) ; 13.8 \mathrm{~V}(\mathrm{C} 4, \mathrm{C} 5) ; 13.5 \mathrm{~V}(\mathrm{C} 2, \mathrm{C} 3) ; 12.7 \mathrm{~V}(\mathrm{C} 2, \mathrm{C} 4) ; 12.0 \mathrm{~V}(\mathrm{H} 11, \mathrm{C} 3) ; 12.0 \mathrm{~V}(\mathrm{H} 12, \mathrm{C} 3) ; \\ & 4.0 \mathrm{C}(\mathrm{C} 4) ; 3.27 \mathrm{~V}(\mathrm{C} 4, \mathrm{C} 6) ; 3.0 \mathrm{C}(\mathrm{C} 3) \end{aligned}$ |
| $\mathrm{V}(\mathrm{C} 4, \mathrm{H})$ | 2.16 | 0.71 | 0.33 | 74.83 | 1.000 | $18.9 \mathrm{~V}(\mathrm{C} 4 . \mathrm{C} 5) ; 14.5 \mathrm{~V}(\mathrm{C} 3, \mathrm{C} 4) ; 13.7 \mathrm{~V}(\mathrm{C} 2, \mathrm{C} 4) ; 13.0 \mathrm{~V}(\mathrm{C} 4, \mathrm{C} 6) ; 10.9 \mathrm{C}(\mathrm{C} 4)$ |
| V(C4,C5) | 1.64 | 0.98 | 0.60 | 20.41 | 0.915 | $\begin{aligned} & 13.7 \mathrm{~V}(\mathrm{H}, \mathrm{C} 4) ; 13.5 \mathrm{~V}(\mathrm{H}, \mathrm{C} 5) ; 13.0 \mathrm{~V}(\mathrm{C} 5, \mathrm{C} 6) ; 10.9 \mathrm{VC} 5, \mathrm{C} 8) ; 10.3 \mathrm{~V}(\mathrm{C} 3, \mathrm{C} 4) ; 8.8 \mathrm{~V}(\mathrm{C} 4, \mathrm{C} 6) ; \\ & 5.8 \mathrm{~V}(\mathrm{C} 2, \mathrm{C} 4) ; 5.2 \mathrm{C}(\mathrm{C} 4) ; 3.8 \mathrm{C}(\mathrm{C} 5) \end{aligned}$ |
| V(C4, C6) | 0.83 | 0.63 | 0.76 | 10.28 | 0.810 | $\begin{aligned} & 14.9 \mathrm{~V}(\mathrm{C} 2, \mathrm{C} 4) ; 14.7 \mathrm{~V}(\mathrm{H}, \mathrm{C} 4) ; 13.7 \mathrm{~V}(\mathrm{C} 4, \mathrm{C} 5) ; 12.0 \mathrm{~V}(\mathrm{C} 5, \mathrm{C} 6) ; 11.0 \mathrm{~V}(\mathrm{H}, \mathrm{C} 6) ; 9.4 \mathrm{~V}(\mathrm{C} 6, \mathrm{C} 7) \\ & 3.8 \mathrm{~V}(\mathrm{C} 3, \mathrm{C} 4) ; 3.7 \mathrm{C}(\mathrm{C} 4) ; 2.4 \mathrm{C}(\mathrm{C} 6) \end{aligned}$ |
| V(C5,C6) | 2.02 | 1.06 | 0.52 | 26.12 | 0.947 | $\begin{aligned} & 15.5 \mathrm{~V}(\mathrm{H}, \mathrm{C} 6) ; 15.3 \mathrm{~V}(\mathrm{H}, \mathrm{C} 5) ; 12.8 \mathrm{~V}(\mathrm{C} 6, \mathrm{C} 7) ; 12.4 \mathrm{~V}(\mathrm{C} 5, \mathrm{C} 8) ; 12.0 \mathrm{~V}(\mathrm{C} 4 . \mathrm{C} 5) ; 7.1 \mathrm{~V}(\mathrm{C} 4, \mathrm{C} 6) \text {; } \\ & 5.9 \mathrm{C}(\mathrm{C} 6) ; 5.4 \mathrm{C}(\mathrm{C} 5) \end{aligned}$ |
| V(C6, 7 7) | 1.99 | 1.03 | 0.52 | 20.19 | 0.958 | $\begin{aligned} & 15.6 \mathrm{~V}(\mathrm{H}, \mathrm{C} 6) ; 14.9 \mathrm{~V}(\mathrm{H} 16, \mathrm{C} 7) ; 14.9 \mathrm{~V}(\mathrm{H} 17, \mathrm{C} 7) ; 13.1 \mathrm{~V}(\mathrm{C} 5, \mathrm{C} 6) ; 11.7 \mathrm{~V}(\mathrm{C} 1, \mathrm{C} 7) ; 6.1 \mathrm{C}(\mathrm{C} 6) ; \\ & 5.7 \mathrm{~V}(\mathrm{C} 4, \mathrm{C} 6) ; 5.0 \mathrm{C}(\mathrm{C} 7) \end{aligned}$ |
| V(C6,H) | 2.14 | 0.66 | 0.31 | 78.54 | 1.000 | 25.2V(C5,C6); 24.5V(C6,C7); 11.7C(C6); 10.6V(C4,C6) |

that these four basins lose their union with $\mathrm{V}(\mathrm{C} 4, \mathrm{H})$ at an ELF value of 0.712 and $\mathrm{V}(\mathrm{C} 4, \mathrm{C} 5)$ and $\mathrm{V}(\mathrm{C} 4, \mathrm{C} 6)$ have a bifurcation value of 0.702 , the $\mathrm{V}(\mathrm{C} 2, \mathrm{C} 4)$ and $\mathrm{V}(\mathrm{C} 4, \mathrm{C} 6)$ union (the bifurcation value is 0.786 and is close to the value of the $\mathrm{V}(\mathrm{C} 4,-$ C6) attractor (0.810)) can be considered "chemically significant" in this ELF analysis. In fact, the $\mathrm{V}(\mathrm{C} 2, \mathrm{C} 4)$ and $\mathrm{V}(\mathrm{C} 4, \mathrm{C} 6)$ disynaptic basins merged into a single trisynaptic basin involving $\mathrm{C} 2, \mathrm{C} 4$, and C 6 when the $\mathrm{C} 4-\mathrm{C} 6$ distance was increased to $1.78 \AA$. Yet the energy of $\mathbf{1}$ only increased by $0.38 \mathrm{kcal} \mathrm{mol}^{-1}$. At a C4-C6 distance of $1.87 \AA$ the connection to C 6 was lost and the trisynaptic basin was reduced to a disynaptic basin between C2 and C4. In going from 1-opt to the $1.87 \AA$ geometry, the total increase in energy was $1.42 \mathrm{kcal} \mathrm{mol}^{-1}$ ! At the Becke3LYP/6-311G(d,p) level the disynaptic V(C4,C6) basin merged with $\mathrm{V}(\mathrm{C} 2, \mathrm{C} 4)$ to form a trisynaptic basin when the $\mathrm{C} 4-\mathrm{C} 6$ distance was $1.78 \AA\left(\Delta E_{\mathrm{T}}\right.$ was $\left.+0.13 \mathrm{kcal} \mathrm{mol}^{-1}\right)$ and the trisynaptic basin collapsed into $\mathrm{V}(\mathrm{C} 2, \mathrm{C} 4)$ at $1.87 \AA$. $\Delta E_{\mathrm{T}}$ was $0.76 \mathrm{kcal} \mathrm{mol}^{-1}$ relative to 1-opt at the Becke3LYP/ 6-311G(d,p) level. We observed this dependence on functional while studying the 2-norbornyl cation. ${ }^{10}$ In our view, the V(C4,C6) disynaptic basin does not identify a bonifide covalent bonding interaction. On the whole, the C4,C6 interaction appears to be ionic in character, in accord with the fact that the $\mathrm{V}(\mathrm{C} 4,-$ C6) basin is closer to C 4 (the attractor is $0.777 \AA$ from C4 and $1.002 \AA$ from C6) and the $\mathrm{C}(\mathrm{C} 4)-\mathrm{V}(\mathrm{C} 4, \mathrm{C} 6)-\mathrm{C}(\mathrm{C} 6)$ angle of $144.8^{\circ}$ deviates significantly from the geometric interatomic line.

In going from 1-opt to the $1.62 \AA$ geometry where C 4 and C 6 are connected with a bond path, the population of $\mathrm{V}(\mathrm{C} 4,-$ C6) increased from 0.83 to 1.13 e , the variance increased from 0.63 to 0.79 , the volume increased from 10.28 to 13.53 bohr $^{3}$, and the fluctuation decreased from 0.76 to 0.70 . These data and the fact that the ELF value at the $(3,-3) \mathrm{CP}$ increased from 0.81 to 0.85 indicate a sizable buildup of electronic charge between C 4 and C6. Yet, in going from 1-opt to the 1.62 geometry the energy change was only $0.51 \mathrm{kcal} \mathrm{mol}^{-1}$. V(C4,C6) is disynaptic and it has significant percent contributions to the fluctuation only from C4 and C6 core basins.

In general, homeomorphism is observed between the number and spatial arrangement of the maxima displayed by ELF and the VSCCs of $L(\mathbf{r}) .{ }^{31}$ This was not the case in the $\mathbf{1}$-opt. One disynaptic basin was located between C 4 and C 6 but nearer to C4 than C6, while a VSCC was located within the valence shell of C6 directed to C4. In our case the ELF disynaptic basins around C4 correlate with the $(3,-1)$ critical points in $L(\mathbf{r})$ but


Figure 5. Schematic diagram of the variation of selected critical points as a function of the $\mathrm{C} 4-\mathrm{C} 6$ distance of $\mathbf{1}$.
not the with theVSCCs ( $3,-3 \mathrm{CPs}$ ) of $L(\mathbf{r})$ around C4. Since $L(\mathbf{r})$ exhibits a homeomorphism with the Laplacian of the conditional same-spin pair density, it is the topology of $L(\mathbf{r})$ rather than that of ELF that is to be used in cases where the two fields differ in their predictions. ${ }^{15,31}$

In summary, all carbon atoms of 1-opt except C6 are connected by four bond paths and only four valence shell (3, $-3)$ CPs are found in the Laplacian. Yet, a $(3,-1)$ critical point with a close-to-zero value is found between C 4 and C 6 , showing that there are five $(3,-1) \mathrm{CPs}$ in the Laplacian between C 4 and five other atoms. In line with this result is the finding that there are five disynaptic basins in ELF that connect C4 and other five atoms. Since C4 has four bond paths and only four valence shell $(3,-3)$ CPs are found in Laplacian, we consider 1-opt to be best described as a classical cation that has a high degree of delocalization involving C 4 and C 6 ; it's a $~ \sigma / \pi$ nobond homoconjugated species.

Nature of AIM and ELF Critical Points as a Function of the C4-C6 Distance. The nature of the critical points around C 4 as a function of $\mathrm{C} 4-\mathrm{C} 6$ distance are summarized graphically in Figure 5. There are five bond paths to C 4 when the $\mathrm{C} 4-\mathrm{C} 6$ distance ranged from 1.5 to $1.62 \AA$, while five VSCCs were located around C 4 when the $\mathrm{C} 4-\mathrm{C} 6$ distance ranged between 1.55 and $1.67 \AA$. The nominal cationic center C 6 possessed four VSCCs up to a distance of $1.72 \AA$ and then the one directed at C 4 was lost. When the $\mathrm{C} 4-\mathrm{C} 6$ distance was reduced to less than $1.50 \AA, \mathrm{C} 2$ lost one VSCC and the $\mathrm{C} 2-\mathrm{C} 4$ bond path disappeared. The $(3,-1)$ CPs in $L(\mathbf{r})$ located midway between


Figure 6. Display of (a) the gradient vector field of the electron density of $\mathbf{1}$ in the $\mathrm{C} 4-\mathrm{C} 5-\mathrm{C} 6$ plane when C4-C6 distance is $1.62 \AA$, and (b) the gradient vector field of the electron density in the $\mathrm{C} 4-\mathrm{C} 5-\mathrm{C} 6$ plane of $\mathbf{1}$ substituted with BeH groups at C4 and C5. The bond CPs are labeled with circles. The $(3,+3)$ ring CPs are labeled with an asterisk.
the nuclei and ELF basins have common features and both are not very sensitive to the changes of $\mathrm{C} 4-\mathrm{C} 6$ distance. As seen from the gradient vector field of density in the C4-C5-C6 plane, a curved bond path appears between C 4 and C 6 at a $\mathrm{C} 4-$ C6 distance of $1.62 \AA$ (Figure 6a). At this point the C4-C6 DI is 0.711 . This result is reminiscent of what we found for the 2-norbornyl cation. The densities at the bond critical point and at the ring critical point are virtually identical (1.81 and 1.80 e $\AA^{-3}$, respectively). The Laplacian at these two points is close to zero. From 1.697 to $1.620 \AA$, the total energy only increased by $0.51 \mathrm{kcal} \mathrm{mol}^{-1}$. At this point the covalent bonding between C4 and C6 is marginal. While there is a bond path and five VSCCs are found around C4, the bond path dwells away from atomic connection line and the bond CP is very close to the ring CP , an indication that these CPs are close to the point of anhilation as was seen for $4,5-(\mathrm{BeH})_{2}-\mathbf{1}$ (Figure 6b). The properties at the selected bond critical points and the properties of the selected ELF basins are listed in Table 7 and Table 8 that are included as Supporting Information.

## Conclusions

The 6-tricyclo[3.2.1.0 $0^{2,4}$ ]octyl cation is a classical species that exhibits a $\sigma / \pi$ no-bond homoconjugation between C 4 and C6. It has no pentacoordinated carbon atoms and the bonding may have an ionic component. Only when the C4-C6 distance is fixed in the range $1.50-1.62 \AA$ does it become a pentacoordinate (nonclassical) species in which C4 is connected to C2, C3, C5, C6, and H13 with bond paths. For polycyclic species such as 1, it appears that a delocalization index of approximately 0.7 and an internuclear distance of $1.6 \AA$ are required for a bond path to materialize between remote carbons. The 6 -tricyclo[3.2.1.0 ${ }^{2,4}$ ]octyl cations substituted with Li and BeH groups at C 4 and C 5 are pentacoordinate species.

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Supporting Information Available: Table 2 giving selected geometry parameters, total energies, and relative energies of $\mathbf{1}$ as a function of the $\mathrm{C} 4-\mathrm{C} 6$ distance and Tables 7 and 8 listing properties at the selected bond critical points and the properties of the selected ELF basins. This material is available free of charge via the Internet at http://pubs.acs.org.

## References and Notes

(1) (a) Wiberg, K. B.; Wenzinger, G. R. J. Org. Chem. 1965, 30, 2278. (b) Colter, A. K.; Musso, R. C. J. Org. Chem. 1965, 30, 2462.
(2) Berson, J. A.; Bergman, R. G.; Clarke, G. M.; Wege, D. J. Am. Chem. Soc. 1969, 91, 5601.
(3) Werstiuk, N. H.; Wang, Y. G. ARKIVOC, in press.
(4) Haywood-Farmer, J. Chem. Rev. 1974, 74, 315.
(5) Cremer, D.; Childs, R. L.; Kraka, E. In The Chemistry of the cyclopropyl group; Rappoport, Z., Ed.; John Wiley \& Sons Ltd: New York, 1995; Vol. 2, pp 339-410.
(6) Cecchi, P.; Pizzabiocca, A.; Renzi, G.; Grandinetti, F.; Sparapani, C.; Buzek, P.; Schleyer, P von R.; Speranza, M. J. Am. Chem. Soc. 1993, 115, 10338.
(7) Szabó, K. J.; Cremer, D. J. Org. Chem. 1995, 60, 2257.
(8) Werstiuk, N. H.; Muchall, H. M. J. Mol. Struct. (THEOCHEM) 1999, 463, 225.
(9) Muchall, H. M.; Werstiuk, N. H. J. Phys. Chem. A 1999, 103, 6599. (10) Werstiuk, N. H.; Muchall, H. M. J. Phys. Chem. A 2000, 104, 2054.
(11) Werstiuk, N. H.; Muchall, H. M.; Noury, S. J. Phys. Chem. A 2000, 104, 11601.
(12) Bader, R. F. W. Atoms in Molecules-A Quantum Theory; Oxford University Press: Oxford, U.K., 1990.
(13) Bader, R. F. W. J. Phys. Chem. A 1998, 102, 731.
(14) Fradera, X.; Austen, M. A.; Bader, R. F. W. J. Phys. Chem. A 1999, 103, 304.
(15) Molina, J. M.; Dobado, J. A.; Heard, G. L.; Bader, R. F. W.; Sundberg, M. R. Theo. Chem. Acc. 2001, 105, 365.
(16) Bader, R. F. W.; Matta, C. F. A SEA CHANGE.
(17) Becke, A. D.; Edgecombe, K. E. J. Chem. Phys. 1990, 92, 5397.
(18) Silvi, B.; Savin, A. Nature 1994, 371, 683.
(19) Noury, S.; Colonna, A.; Savin, A.; Silvi, B. J. Mol. Struct. (THEOCHEM) 1998, 450, 59.
(20) Savin, A.; Silvi, B.; Colonna, F. Can. J. Chem. 1996, 74, 1088.
(21) Savin, A.; Nesper, R.; Wengert, S.; Fassler, T. F. Angew. Chem., Int. Ed. Engl. 1997, 36, 1809.
(22) Frisch, M. J.; Trucks, G. W.; Schlegel, H. B.; Gill, P. M. W.; Johnson, B. G.; Robb, M. A.; Cheeseman, J. R.; Keith, T.; Petersson, G. A.; Montgomery, J. A.; Raghavachari, K.; Al-Laham, M. A.; Zakrzewski, V. G.; Ortiz, J. V.; Foresman, J. B.; Peng, C. Y.; Ayala, P. Y.; Chen, W.; Wong, M. W.; Andres, J. L.; Replogle, E. S.; Gomperts, R.; Martin, R. L.;

Fox, D. J.; Fox, D. J.; Binkley, J. S.; Defrees, D. J.; Baker, J.; Stewart, J. P.; Head-Gordon, M.; Gonzalez, C.; Pople, J. A. Gaussian 94, Revision B.3; Gaussian, Inc.: Pittsburgh, PA, 1995.
(23) Frisch, M. J.; Trucks, G. W.; Schlegel, H. B.; Scuseria, G. E.; Robb, M. A.; Cheeseman, J. R.; Zakrzewski, V. G.; Montgomery, J. A.; Stratmann, R. E.; Burant, J. C.; Dapprich, S.; Millam, J. M.; Daniels, A. D.; Kudin, K. N.; Strain, M. C.; Farkas, O.; Tomasi, J.; Barone, V.; Cossi, M.; Cammi, R.; Mennucci, B.; Pomelli, C.; Adamo, C.; Clifford, S.; Ochterski, J.; Petersson, G. A.; Ayala, P. Y.; Cui, Q.; Morokuma, K.; Malick, D. K.; Rabuck, A. D.; Raghavachari, K.; Foresman, J. B.; Cioslowski, J.; Ortiz, J. V.; Stefanov, B. B.; Liu, G.; Liashenko, A.; Piskorz, P.; Komaromi, I.; Gomperts, R.; Martin, R. L.; Fox, D. J.; Keith, T.; Al-Laham, M. A.; Peng, C. Y.; Nanayakkara, A.; Gonzalez, C.; Challacombe, M.; Gill, P. M. W.; Johnson, B. G.; Chen, W.; Wong, M. W.; Andres, J. L.; Head-Gordon, M.; Replogle, E. S.; Pople, J. A. Gaussian 98, Revision A.1; Gaussian, Inc.: Pittsburgh, PA, 1998.
(24) Biegler-Konig, F. W.; Bader, R. F. W.; Tang, T.-H. J. Comput. Chem. 1982, 3, 317.
(25) Matta, C. F. AIMDELOC 01, Quantum Chemistry Program Exchange; Indiana University: Bloomington, IN, 2001; QCPE0802.
(26) Noury, S.; Krokidis, X.; Fuster, F.; Silvi, B. TopMod 1997.
(27) Noury, S.; Krokidis, X.; Fuster, F.; Silvi. B. Compu., Chem. 1999, 23, 597.
(28) Pepke, E.; Murray, J.; Lyons, J.; Hwu, T.-Y. SciAn, Version 1.21 Alpha; Supercomputer Computations Research Institute, Florida State University: Tallahassee, FL.
(29) MacDougall, P. J. The Laplacian of The Electronic Charge Distribution. Ph.D. Thesis, McMaster University, 1989.
(30) Calatayud, M.; Andres, J.; Beltran, A.; Silvi, B. Theor. Chem. Acc. 2001, 105, 299.
(31) Bader, R. F. W.; Johnson, S.; Tang, T.-H.; Popelier, P. L. A. J. Phys. Chem. 1996, 100, 15398.


[^0]:    ${ }^{a}$ Unless otherwise noted, the calculation level is Becke3PW91/6-311G(d,p)//Becke3PW91/6-311G(d,p). ${ }^{b}$ The percent localization, $|F(\mathrm{~A}, \mathrm{~A})| /$ $\bar{N}(\mathrm{~A}) \times 100 .{ }^{c}$ For the parent hydrocarbon tricyclo[3.2.1.0 ${ }^{2.4}$ ]octane at the Becke3PW91/6-311G(d,p)//Becke3PW91/6-311G(d,p) level, the localization index and the percent localization at $\mathrm{C} 4, \mathrm{C} 6$, and $\mathrm{H}(\mathrm{C} 4)$ are 3.945 and $65.3 \%, 3.913$ and $65.4 \%$, and 0.428 and $43.5 \%$, respectively. The delocalization index between C 4 and C 6 is 0.057 . ${ }^{d}$ The values in parentheses were obtained with the wave function obtained at the HF/6-311G(d,p)//Becke3PW91/ $6-311 \mathrm{G}(\mathrm{d}, \mathrm{p})$ level. ${ }^{e}$ The values were obtained with the wave function obtained at the $\mathrm{HF} / 6-311 \mathrm{G}(\mathrm{d}, \mathrm{p}) / / \mathrm{HF} / 6-311 \mathrm{G}(\mathrm{d}, \mathrm{p})$ level.

